Int. J. Heat Mass Transfer. Vol. 28, No. 5, pp. 1031–1038, 1985
Printed in Great Britain

0017-9310/85 $3.00 + 0.00

Viscosity induced non-uniform flow in laminar flow heat exchangers

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(*Received* 23 *May* 1984 *and in revised form* 12 *November* 1984)

Abstract—Laminar flow heat exchangers which cool oil in non-interconnected parallel passages can experience non-uniform flows and a reduction in the effective heat exchanger coefficient in a range of Reynolds number which varies with tube length and diameter, tube wall temperature and fluid inlet temperature. The method of predicting the reduction in effective heat transfer coefficient and the range of Reynolds number over which these instabilities exist is presented for a particular oil, Mobil aviation oil 120. Included, also, is the prediction of the effect of radial viscosity variation on the constant property magnitudes of friction and heat transfer coefficient.

INTRODUCTION

HEAT exchangers that operate in the laminar flow regime with non-interconnected parallel passages can depart from predicted performance as a result of non-uniform flow distributions. Variations in the flow distribution among the tubes can be caused by the effect of the fluid viscosity/temperature relation, natural convection and non-uniform passage sizes (from design and manufacturing tolerances). If the exchanger is oriented vertically with the colder side up, the effect of natural convection can be eliminated. Non-uniform passage size can be minimized through quality control. Therefore, the effect of the viscosity/temperature relation on non-uniform flow will be analyzed in this paper.

The viscosity/temperature relation for the Mobil aviation oil 120 used in this analysis is presented in Fig. 1. Viscosity is a strong function of temperature, decreasing as temperature increases. In this analysis it is assumed that other fluid properties such as density and thermal conductivity are uniform at $\rho = 55.0$ lbm ft^{-3} (881.0 kg m^{-3}), $k = 0.08$ BTU hr^{-1} ft^{-1} °F^{-1} (0.138 W m^{-1} °C^{-1}) and $c_p = 0.5$ BTU lbm °F^{-1} (2.09×10^3 J kg^{-1} °C^{-1}). This is a reasonable assumption for oils. Also, it is assumed that pressure drop entrance and exit effects can be neglected. The analysis was performed for flow in tubes. Uniform wall temperature was used assuming the heat transfer coefficient was very much greater on the outside than on the inside of the tube.

When cooling liquids or heating gases, non-uniform flow can exist in the flow passages. For the same overall pressure drop, two discrete flow rates can exist in the same heat eachanger.

This problem has been discussed by Mueller [1], [2] and [3] and Rohsenow [4].

EFFECT OF RADIAL VISCOSITY VARIATION

The fully developed fluid and temperature distributions in a tube are non-parabolic due to the viscosity/temperature relation. This causes a lower velocity of the colder fluid near the tube wall and a higher velocity of the hotter fluid near the centerline compared with the constant property case (i.e. uniform viscosity) with the same flow rate and bulk temperature.

For steady state, fully developed flow and viscosity as a function of temperature, the momentum equation

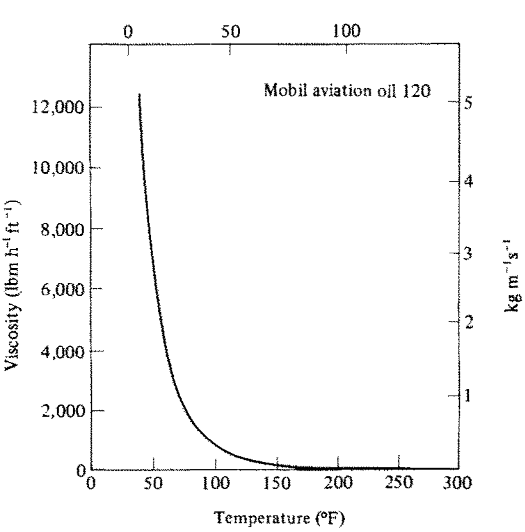

FIG. 1. Viscosity vs temperature relation.

NOMENCLATURE

c_p	specific heat at constant pressure
D	tube diameter
f	friction factor (variable viscosity)
f_{c_p}	friction factor (uniform viscosity evaluated at the bulk temperature)
G	mass flow rate
h_{eff}	effective heat transfer coefficient
k	thermal conductivity
L	tube length
m	exponent for friction factor correction
n	exponent for Nusselt number correction
n_0	number of tubes
Nu_{eff}	effective Nusselt number (variable viscosity)
Nu	Nusselt number (variable viscosity)
Nu_{c_p}	Nusselt number (uniform viscosity at the bulk temperature)
P	pressure along the tube
ΔP	tube pressure drop
r	tube radius, radial direction
Re_b	Reynolds number evaluated at μ_b
T	temperature as function of r or x
T_{avg}	average exit temperature in non-uniform flow condition
T_b	bulk temperature
T_0	wall temperature
T_i	inlet temperature
$T_{uniform}$	average exit temperature for uniform flow condition
u	fluid velocity in x-direction
W	total flow rate
w	individual tube flow rate
x	direction along tube.

Greek symbols

ρ	fluid density
μ	viscosity as function of T
μ_0	viscosity evaluated at T_0
μ_b	viscosity evaluated at T_b.

reduces to

$$\frac{dp}{dx} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right). \tag{1}$$

Neglecting the effects of axial conduction and assuming constant heat flux at the tube wall, the temperature distribution for fully developed conditions is

$$rk\frac{\partial T}{\partial r} = \frac{d}{dx}\int \rho c_p u T r\, dr. \tag{2}$$

An expression is needed to account for the change in friction factor and Nusselt number for the variable properties case (i.e. variable viscosity) compared with the constant properties case (i.e. uniform viscosity). Assume that the effect can be expressed as the ratio of the wall to bulk viscosities raised to an exponent as follows

$$\frac{f}{f_{c_p}} = \left(\frac{\mu_0}{\mu_b}\right)^m \tag{3}$$

$$\frac{Nu}{Nu_{c_p}} = \left(\frac{\mu_0}{\mu_b}\right)^n. \tag{4}$$

f_{c_p} and Nu_{c_p} are based on constant properties at the bulk temperature.

To find exponents m and n, equations (1) and (2) were solved simultaneously using an iterative procedure. An average fluid velocity, wall temperature and parabolic temperature distribution were assumed initially. The fluid viscosity vs temperature relation of Fig. 1 was closely approximated by a piecewise curve fit using a sixth degree polynomial. Based on the assumed temperature distribution, the viscosity $\mu(r)$ was determined. Equation (1) was solved for the fluid velocity based on the viscosity $\mu(r)$. The dp/dx term was adjusted to obtain the desired average fluid velocity. Next, the temperature distribution from equation (2) was determined using the calculated fluid velocity. Then, the friction factor and Nusselt number were calculated. The viscosity $\mu(r)$ was determined for the revised temperature distribution and the procedure was repeated until the calculated friction factors and Nusselt numbers agreed within less than 1%.

The results of this analysis showed that the exponents m and n varied as a function of wall and bulk temperature as shown in Figs. 2 and 3. The exponents were invariant to changes in the average fluid velocity.

This problem has not been investigated extensively for various μ vs T relations. Recommendations found in the literature are $m = 0.5$ and $n = 0.14$.

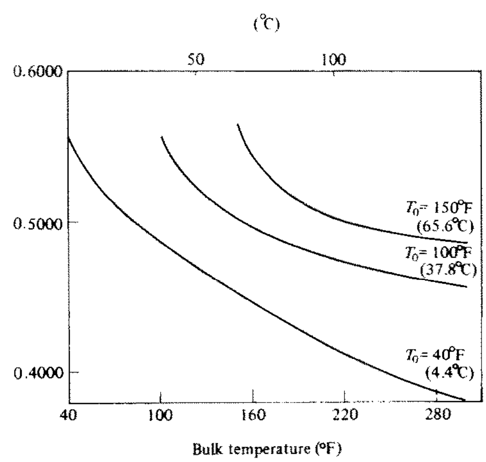

FIG. 2. Exponent m.

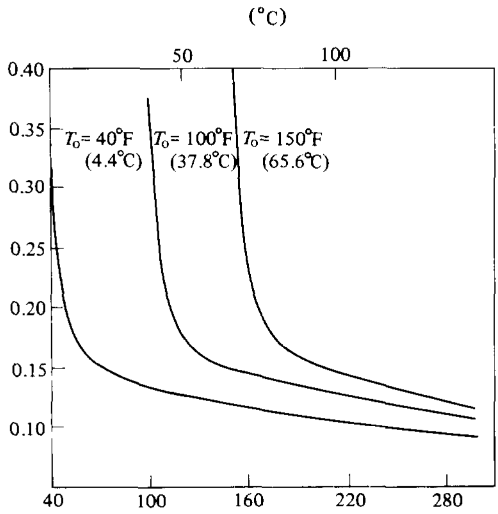

FIG. 3. Exponent n.

SINGLE TUBE PRESSURE DROP

Having quantified the radial effects of the viscosity/temperature relation on friction factor and Nusselt number using the ratio of the wall to bulk viscosities and the exponents m and n, an analysis is performed on a heat exchanger tube. An energy balance at the tube wall for uniform wall temperature yields

$$wc_p \, dT = h\pi D(T-T_0) \, dx \tag{5}$$

or

$$\frac{dT}{T-T_0} = \frac{\pi k}{wc_p}\left(\frac{hD}{k}\right) dx. \tag{6}$$

Preliminary calculations showed that the fluid flow was fully developed a short distance from the tube entrance and entrance effects could be neglected. However, entrance effects on the temperature distribution had to be taken into account because of the slowly developing temperature distribution. The following equation was suggested by Kays [5] to account for variation in the Nusselt number in the entrance region of tubes for uniform wall temperature

$$Nu = 3.66 + \frac{0.0668 \dfrac{4}{\pi x} \dfrac{wc_p}{k}}{1+0.04\left[\dfrac{4}{\pi x}\dfrac{wc_p}{k}\right]^{2/3}}. \tag{7}$$

Therefore, the equation to determine the temperature drop along the heat exchanger is given by combining equations (4), (6) and (7):

$$\frac{\Delta T}{T-T_0} = \left(\frac{\mu_0}{\mu_b}\right)^n \times\left(3.66 + \frac{0.0668 \dfrac{4}{\pi x} \dfrac{wc_p}{k}}{1+0.04\left[\dfrac{4}{\pi x}\dfrac{wc_p}{k}\right]^{2/3}}\right)\frac{\pi k}{wc_p}\Delta x. \tag{8}$$

Equation (8) includes the radial effects and the developing temperature distribution effects. The temperature change along the tube was solved by iteration of values of bulk viscosity, local temperature and exponent n. The temperature change and bulk viscosity along the tube for various flow rates is shown in Figs. 4 and 5 for a 0.25-in (0.63-cm) diameter tube wall temperature of 40°F (4.4°C).

It can be shown that the pressure drop for an increment dx along a tube is given by

$$\frac{dp}{dx} = \frac{64}{Re}\frac{1}{D}\frac{G^2}{2\rho}. \tag{9}$$

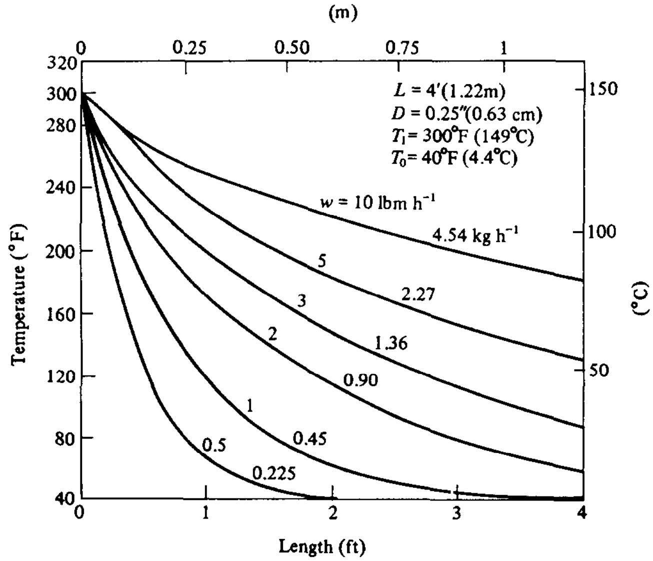

FIG. 4. Temperature variation along exchanger.

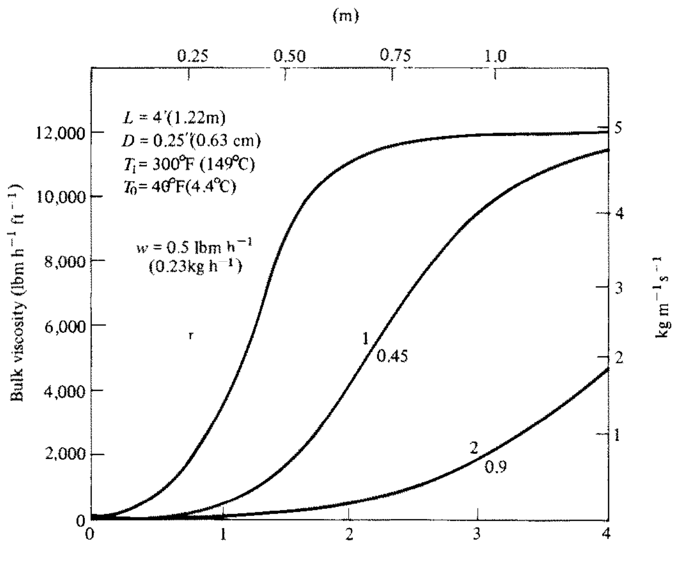

FIG. 5. Viscosity variation along exchanger.

Equation (9) is multiplied by equation (3) to account for radial effects on the friction factor

$$\frac{dp}{dx} = \frac{64}{Re_b}\left(\frac{\mu_0}{\mu_b}\right)^m \frac{1}{D}\frac{G^2}{2\rho}. \tag{10}$$

In integral form equation (10) becomes

$$\Delta P = \frac{128}{\pi\rho D^4} w \int_0^L \mu_0^m \mu_b^{1-m}\, dx. \tag{11}$$

The curves of pressure drop vs flow rate for various lengths of tube were calculated (Fig. 6) for $D = 0.25$ in (0.63 cm), $T_0 = 40°F$ (4.4°C) and $T_i = 300°F$ (149°C). It is observed that ΔP goes through a maximum, then a minimum, followed by a continuous rise as the flow rate is increased. From equation (11), $\Delta P \sim D^{-4}$; therefore ΔP vs w curves for other tube diameters would look like those of Fig. 6 but be displaced vertically proportional to D^{-4}.

MULTI-TUBE HEAT EXCHANGER PERFORMANCE

These curves are for single tubes but may be used to predict the flow maldistribution and resulting effective Nusselt number for multi-tube heat exchangers. This exchanger performance is determined by how the total flow is supplied—constant flow pump (gear pump), constant head source or variable head-flow pump (centrifugal pump).

Constant flow pump

Consider the case of a heat exchanger with only five tubes for simplicity $L = 4$ ft (1.22 m), $D = 0.25$ in (0.63 cm). Non-uniform flow can exist as shown in Fig. 7. Here shell side h is very much greater than tube inside h; therefore tube wall temperature is taken to be essentially uniform. For an inlet flow rate of 11.7 kg h^{-1} (25.75 lbm h^{-1}), average flow rate of 2.3 kg h^{-1} (5.15

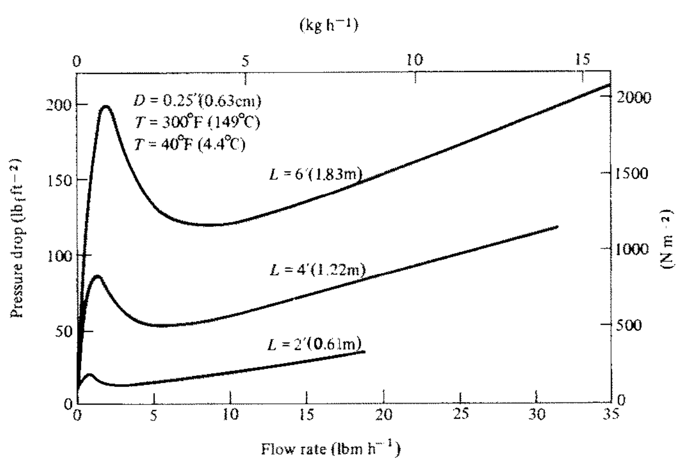

FIG. 6. Pressure drop vs flow rate.

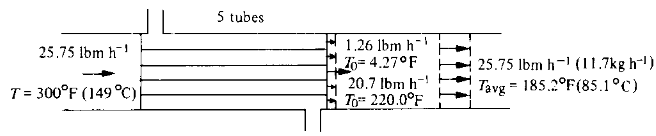

FIG. 7. Five tube heat exchanger.

lbm h^{-1}) per tube, point *d* of Fig. 9, and inlet temperature of 300°F (149°C) and wall temperature of 40°F (4.4°C), four of the tubes have a flow rate of 1.26 lbm h^{-1} (0.571 kg h^{-1}) with an exit temperature of 42.7°F (5.9°C) and one tube has a higher flow rate of 20.7 lbm h^{-1} (9.4 kg h^{-1}) (points a and l of Fig. 9) with an exit temperature of 220.0°F (104.4°C). The average bulk temperature at the exchanger outlet is 185.2°F (85°C). Figure 8 shows a plot of temperature along the exchanger for one tube at higher flow and four tubes with lower flow. The average exit temperature, T_{avg}, is the summation of the individual tube exit temperature weighted by the percentage of flow through each tube. This is the temperature that would be measured at the heat exchanger exit after mixing the five streams. If it were not known that this maldistribution existed and it were assumed that the flow was uniform in the five tubes at one-fifth of the total flow, the effective Nusselt number would be determined using T_{avg} as follows

$$Nu_{eff} = \frac{h_{eff}D}{k}$$
$$h_{eff} = \frac{c_p W}{n_0 \pi D L} \log\left(\frac{T_0 - T_i}{T_0 - T_{avg}}\right). \tag{12}$$

For this case $h_{eff} = 5.72$ BTU h^{-1} ft^{-2} $°F^{-1}$ (32.49 W m^{-2} $°C^{-1}$) and $Nu_{eff} = 1.49$. Also shown in Fig. 8 is the temperature along the exchanger and the exit temperature, $T_{uniform}$, if the tubes, in fact, had uniform flow. For this case $h_{eff} = 10.37$ BTU h^{-1} ft^{-2} $°F^{-1}$ (58.87 W m^{-2} $°C^{-1}$) and $Nu_{eff} = 2.70$. Since $T_{avg} > T_{uniform}$, the Nusselt number (and heat exchanger performance) is lower when non-uniform flows exist.

An enlarged view of the region of non-uniform flows is shown in Fig. 9. On Fig. 9 curve c–d at any ΔP level represents the average flow rate with four tubes having a flow rate on curve u–a and one tube on curve m–l, the stable flow curves. Then $5w_{cd} = 4w_{ua} + w_{ml}$. Curves s–f, g–h and o–j are similar curves for flow splits of 3–2, 2–3 and 1–4 tubes on curves u–a and m–l. As the total flow rate is increased, uniform flow exists through the peak at point a and on to point b. At point b the pressure drop decreases suddenly to point c where one tube has a higher flow rate while the other four tubes have lower flow rates. The total flow rate is: $5w_b = 5w_c = 4w_u + w_m$. As flow is further increased, flow increases in each of the tubes (along the pressure drop curve) with the average flow rate shown as line c–d. Point d is the point depicted in Figs. 7 and 8. As flow is increased slightly above the flow corresponding to point d, the pressure drop falls from points d to e and now two tubes have higher flow rates and three tubes have lower flow rates. As the flow is increased, the flow in each tube increases with the average flow rate shown on the line e–f. This process continues until uniform flow in all five tubes occurs at point k. In the case of infinite tubes, the average flow rate would follow the line a–l with flow in the tubes corresponding to the flow at either point a—

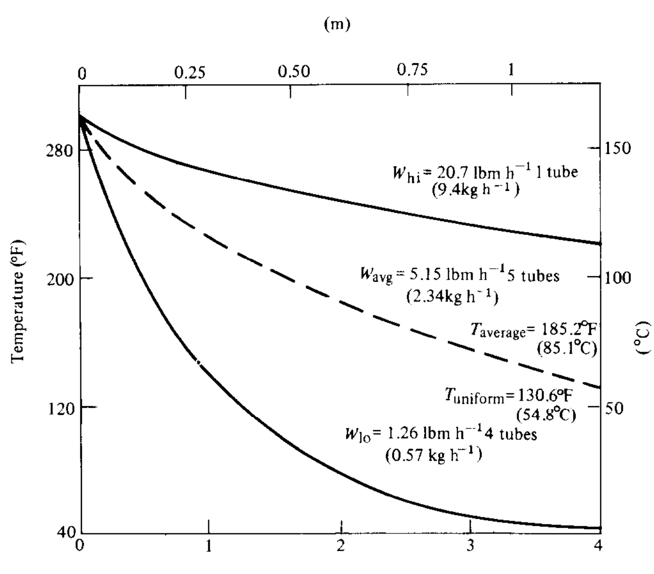

FIG. 8. Temperature distribution for five tube exchanger.

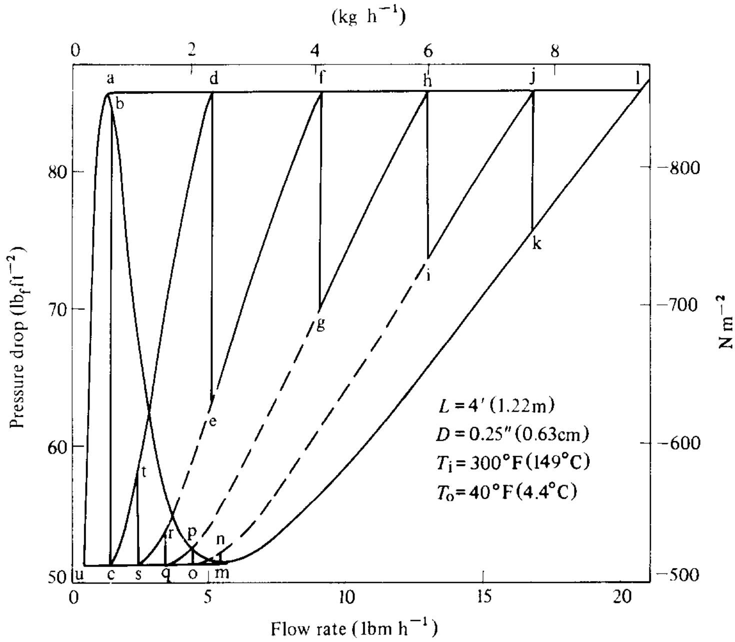

FIG. 9. Flow maldistribution in five tube exchanger.

lower flow rate, or l—higher flow rate. The mix of tubes with higher and lower flow rates is determined by the average fluid flow rate.

For the case of decreasing flow rate, non-uniform flows also occur. As flow is decreased to point m the pressure drop decreases with all tubes having uniform flow. The pressure drop suddenly increases from points m to n at which one tube has lower flow and four tubes have higher flow. As flow is decreased further, flow in each tube decreases with the average flow rate given by line n–o. Additional tubes 'pop-over' to lower flow at points o, q, s and c. All the tubes have the same flow rate at point b. In the case of infinite tubes the average flow rate would follow line m–u with flow in the tubes corresponding to the flow at either point u—lower flow rate, or m—higher flow rate.

Figure 10 shows the effects of non-uniform flow on the Nusselt number as a function of Reynolds number for the case of five tubes. The solid line is for increasing flow and the dashed line is for decreasing flow. The Nusselt number is reduced significantly between points b and k for increasing flow and between points m and b for decreasing flow. These are the regions of the curve where non-uniform flow exists.

For the case of infinite tubes, the effect of non-uniform flow on exchanger performance is shown in Figs. 11 and 12. As the tube diameter is decreased, the region of non-uniform flow is extended to higher Reynolds numbers but the Nusselt number decreases less. Increasing the tube length has an effect on exchanger performance similar to that of decreasing the tube diameter.

Constant head source

For the constant head source, as the pressure drop is increased beyond point a of Fig. 9, the average flow rate

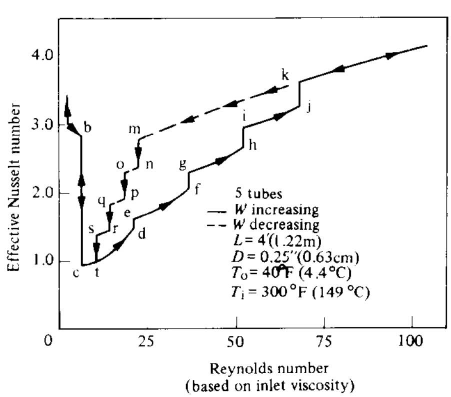

FIG. 10. Five tube exchanger performance.

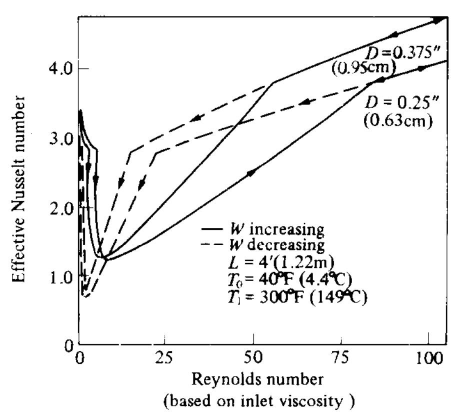

FIG. 11. Effect of diameter on performance for an infinite number of tubes.

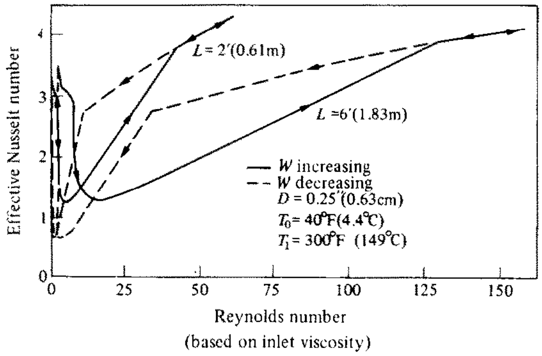

FIG. 12. Effect of length on performance for an infinite number of tubes.

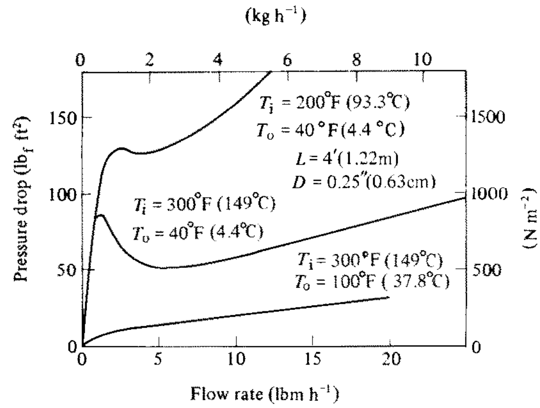

FIG. 14. Effect of temperature levels on pressure drop curves.

will increase to point l regardless of the number of tubes. With continual increase in constant head, operation cannot be sustained at flow rates between points a and l. For the case of continuously decreasing constant head, all tubes remain operating along line l to m. Then the flow drops to point u in all tubes.

Variable head-flow pump

For a centrifugal pump source with the pump characteristic shown as lines a–b and c–d in Fig. 13, non-uniform flows can also exist. The situation is much the same as for the case of a constant head source except that the average flow follows the pump characteristic instead of the constant head line. As pump speed is increased above the point a all tubes shift to operation at point b. The exchanger cannot operate between points a and b at the higher speed nor between points c and d at the lower speed.

DISCUSSION

Variations in the tube wall and inlet temperature can affect the values of exponents m and n (Figs. 2 and 3).

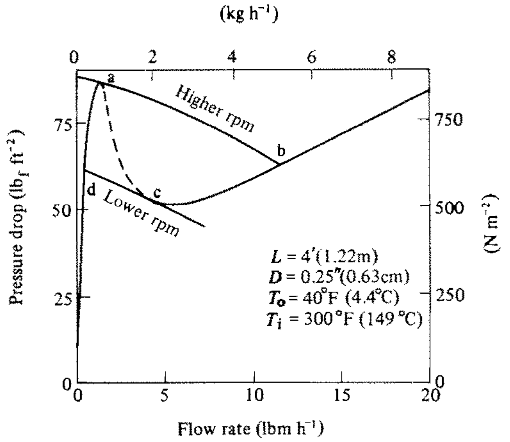

FIG. 13. ΔP vs w for variable head-flow pump for infinite number of tubes.

Together with the μ vs T relation (Fig. 1), changes in exponents m and n can cause differences in the shape of the ΔP vs w curve determined by equation (11). The ΔP vs w curves for a single tube, $L = 4$ ft (1.22 m), $D = 0.25$ in (0.63 cm), are plotted in Fig. 14 for different combinations of wall and inlet temperature. The tube with $T_0 = 40°F$ (4.4°C) and $T_i = 300°F$ (149°C) exhibits a region in which non-uniform flow can exist. If T_i is changed from 300°F (149°C) to 200°F (93°C), there is only a slight dip in the curve and the region of non-uniform flow is narrower. If T_0 is changed from 40°F (4.4°C) to 100°F (37.8°C) with $T_i = 300°F$ (149°C), no region of non-uniform flow exists.

Shown in Fig. 9 are the paths of operation as flow is increased or decreased with a gear pump and a constant head tank. The path of operation is shown for the example of a five tube exchanger and an infinite number tubes. The flow path for a typical centrifugal pump supply is shown in Fig. 13. The resulting effective overall performance Nusselt numbers are shown in Figs. 10, 11, and 12.

Results were similar for calculations done using another oil, Mobil automotive oil HD-30.

The ΔP vs flow curves here are similar to those for forced convection boiling which can result in instabilities due to the compressibility in the boiling system. Here with single phase liquid there should be no instabilities if there is no compressible volume between the pump and the heat exchanger.

CONCLUSIONS

Laminar flow, parallel passage heat exchangers can experience maldistribution of flow and a reduction in the effective heat transfer coefficient in a range of Reynolds number which varies with length, tube diameter and the levels of the temperature—wall and inlet.

For Mobil aviation oil 120 with the μ vs T relation of Fig. 1, the exponents m and n in the viscosity correction ratios, equations (3) and (4), are shown to vary with both wall and bulk temperatures, see Figs. 2 and 3.

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ECOULEMENT NON UNIFORME INDUIT PAR LA VISCOSITE DANS DES ECHANGEURS DE CHALEUR A ECOULEMENT LAMINAIRE

Résumé—Des échangeurs de chaleur à écoulement laminaire avec huile froide dans des passages parallèles non interconnectés peuvent être le siège d'écoulements non uniformes avec une réduction du coefficient effectif d'échange, dans un domaine de nombre de Reynolds qui varie avec la longueur et le diamètre du tube, la température pariétale du tube et la température d'entrée du fluide. La méthode de calcul de la réduction du coefficient de transfert effectif et du domaine de nombre de Reynolds dans lequel ces instabilités existent est présenté pour une huile particulière, l'huile d'aviation Mobil 120. On inclut aussi l'effet de la variation radiale de viscosité sur la grandeur des propriétés des coefficients de frottement et de transfert thermique.

UNGLEICHMÄSSIGE STRÖMUNG AUF GRUND DER ZÄHIGKEIT BEI LAMINARER STRÖMUNG IN WÄRMETAUSCHERN

Zusammenfassung—In Wärmetauschern, in denen Öl bei laminarer Strömung in getrennten, parallelen Kanälen gekühlt wird, kann ungleichmäßige Durchströmung auftreten, was zu einer Verringerung des Wärmeübertragungsvermögens führt. Dies geschieht in einem Bereich der Reynolds-Zahl, der von der Rohrlänge, dem Rohrdurchmesser, der Rohrwandtemperatur und von der Eintrittstemperatur des Fluids abhängt. Es wird eine Methode vorgestellt, mit der man diese Verringerung des Wärmeübertragungsvermögens und den Bereich der Reynolds-Zahl, in dem dies auftritt, vorhersagen kann. Sie wird für ein spezielles Öl, Mobil aviation 120, gezeigt. Mit der Methode wird auch der Einfluß der radialen Änderung der Viskosität auf die mit konstanten Stoffwerten ermittelten Werte der Reibung und des Wärmeübergangskoeffizienten erfaßt.

ОБУСЛОВЛЕННОЕ ВЯЗКОСТЬЮ НЕОДНОРОДНОЕ ТЕЧЕНИЕ В ТЕПЛООБМЕННИКАХ С ЛАМИНАРНЫМ РЕЖИМОМ ТЕЧЕНИЯ

Аннотация—Рассматриваются теплообменники с ламинарным режимом течения, в которых происходит охлаждение масла в системе параллельных несоединенных между собой труб. Показано, что в некотором диапазоне чисел Рейнольдса, который определяется длиной и диаметром трубы, ее температурой и температурой жидкости на входе, может иметь место неустойчивость течения и снижение эффективного коэффициента теплообмена. Представлен метод расчета снижения эффективного теплообмена и диапазона изменения числа Рейнольдса, в котором существует неустойчивость течения, для авиационного масла. Также представлен расчет влияния изменения вязкости в радиальном направлении на величины коэффициентов трения и теплообмена.